Verifying Trigonometric Identities

We have already seen the following identities:

Reciprocal Identities:

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent in Terms of Sine and Cosine:

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

The Fundamental Identity:

$$\sin^2 x + \cos^2 x = 1$$

If we divide each term of the fundamental identity by $\sin^2 x$ or $\cos^2 x$, we can derive two more identities. These are called Pythagorean Identities because they are related to the Pythagorean Theorem:

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$1 + \cot^2 x = \csc^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

Simplifying Expressions

We can use the identities above to simplify trigonometric expressions. One of the most common strategies is to start by rewriting the expression in terms of sines and/or cosines, then simplify from there.

Examples:

$$\frac{\tan x}{\sec x} \qquad \sin x + \cot x \cos x \qquad \frac{\tan x \csc x}{\sec x}$$

Verifying Identities

It is often necessary to determine whether two expressions are equivalent to each other. We can use the approaches from the previous section to verify whether equations are identities.

Multiplying and Factoring Polynomials Involving Trigonometric Functions

We must often multiply binomials or factor trinomials involving trigonometric functions when we verify identity.

Examples:

Multiply $(1+\tan x)(1-\tan x)$

Multiply $(2\sin x+1)^2$

Factor $\sec^2 x-\tan^2 x$

Factor $\sin^2 x+\sin x-2$

A General Strategy for Verifying Identities

1. Work on the more complicated side first.
2. Rewrite the side you are working with in terms of sines and cosines only.
3. Multiply the numerator and denominator of one rational expression by either the numerator or denominator of the other.
4. Write a single rational expression as a sum of two rational expressions.
5. Combine a sum of two rational expressions into a single rational expression.
6. If both sides simplify to a third expression, then the equation is an identity.

Examples:

Verify that $1+\sec x\sin x\tan x=\sec^2 x$ is an identity.

Prove that $\dfrac{\cos\alpha}{1-\sin\alpha}=\dfrac{1+\sin\alpha}{\cos\alpha}$ is an identity.

Prove that $\dfrac{\csc x - \sin x}{\sin x} = \cot^2 x$ is an identity.

Prove that $-2\cot^2 x = \dfrac{1}{1-\sec x} + \dfrac{1}{1+\sec x}$ is an identity.

Prove that $\dfrac{1-\sin^2 t}{1+\csc t} = \dfrac{1-\sin t}{\csc t}$ is an identity.

Show that $\dfrac{1-\cos^2 t}{-\sin t} = -\tan t \cos t$ is an identity.

*See list of Pythagorean Conjugates on page 91 of your textbook.